Simultaneous Spatial and Temporal Assignment for Fast UAV Trajectory Optimization using Bilevel Optimization

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Motivation

- Time allocation and spatial assignment of each waypoint affect the quality of trajectories to the great extents.
- With equality constraints only, the quadratic programing (QP) has a much better performance in computational efficiency.
- For a QP, analytical gradient can be efficiently obtained through implicit differentiation, providing convenient tool for solving bilevel optimization problems.

Formulation

Main problem:
Find time allocation $\boldsymbol{T}$, waypoints $\boldsymbol{\xi}$, and associated polynomial coefficients $\boldsymbol{\sigma}$ of the trajectory and minimize the quadratic cost.

$$\underset{\boldsymbol{\sigma},\boldsymbol{\xi},\boldsymbol{T}\in\mathcal{T}}{\text{minimize}} \quad J(\boldsymbol{\sigma},\boldsymbol{T}) = \boldsymbol{\sigma}^\top P(\boldsymbol{T})\boldsymbol{\sigma} + \boldsymbol{\sigma}^\top \boldsymbol{q}(\boldsymbol{T})$$

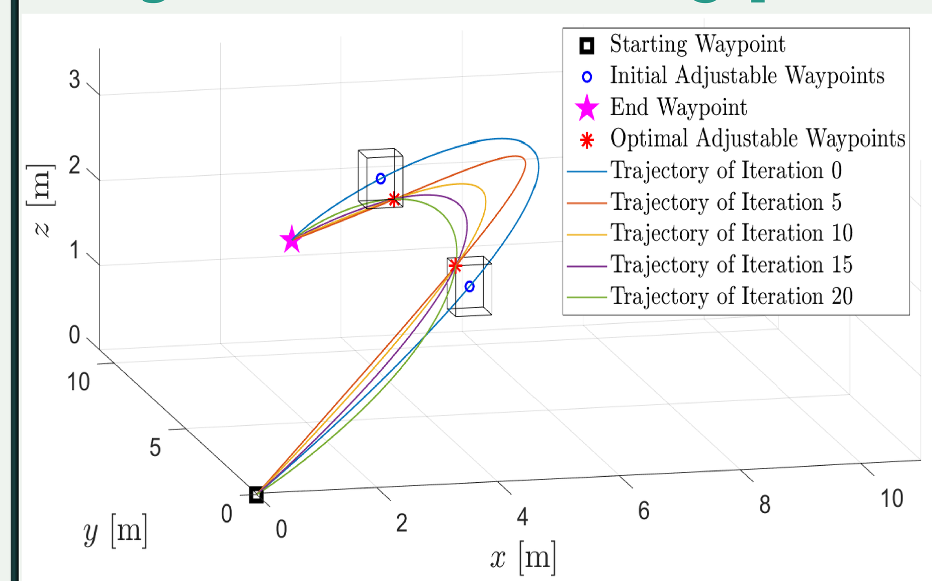
$$\text{subject to} \quad R\boldsymbol{\xi} \preceq \boldsymbol{s}, \quad \text{(Waypoints in safe sets \& dynamical constraints)}$$

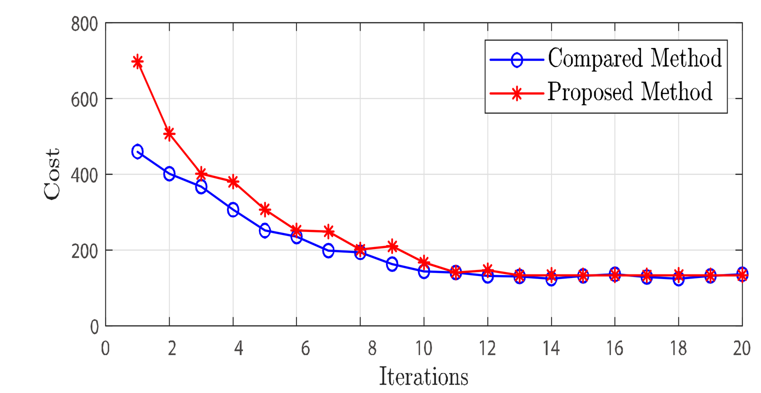
$$C(\boldsymbol{T})\boldsymbol{\sigma} = \boldsymbol{\xi}, \quad \text{(Trajectory passing the waypoints)}$$

$$A(\boldsymbol{T})\boldsymbol{\sigma} = \boldsymbol{b}. \quad \text{(Continuity condition)}$$

Simulation Results

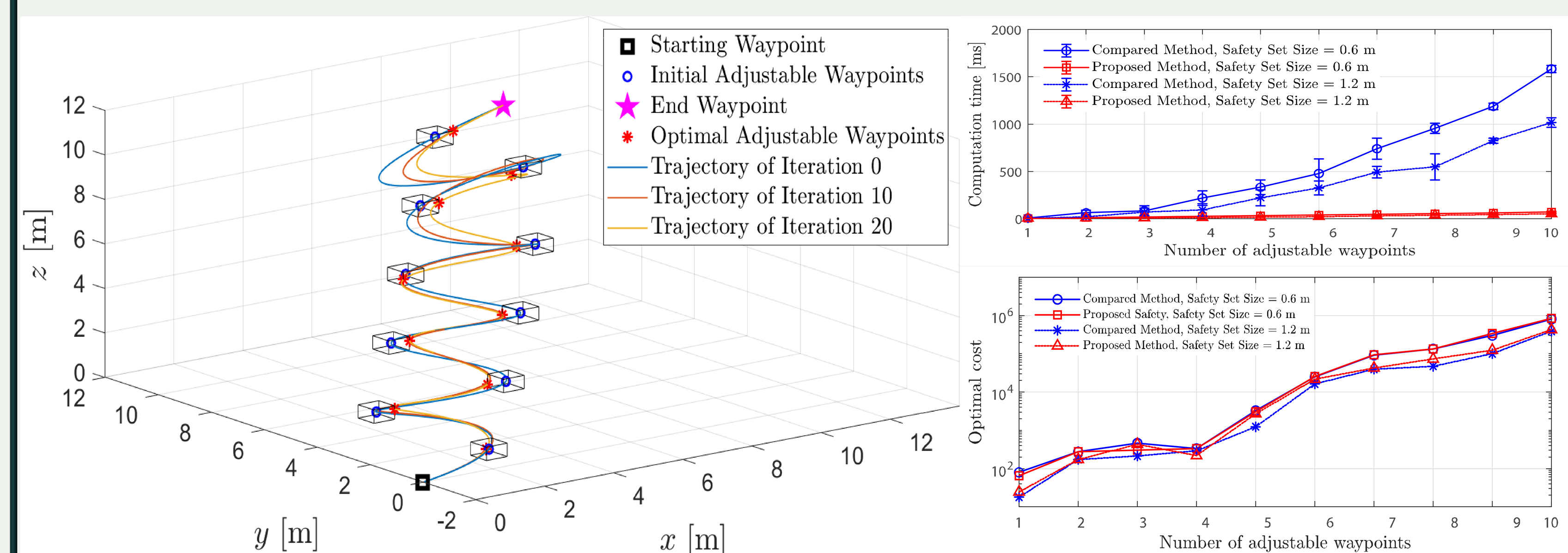
Case 1. Trajectory Planning with Two Adjustable Waypoints



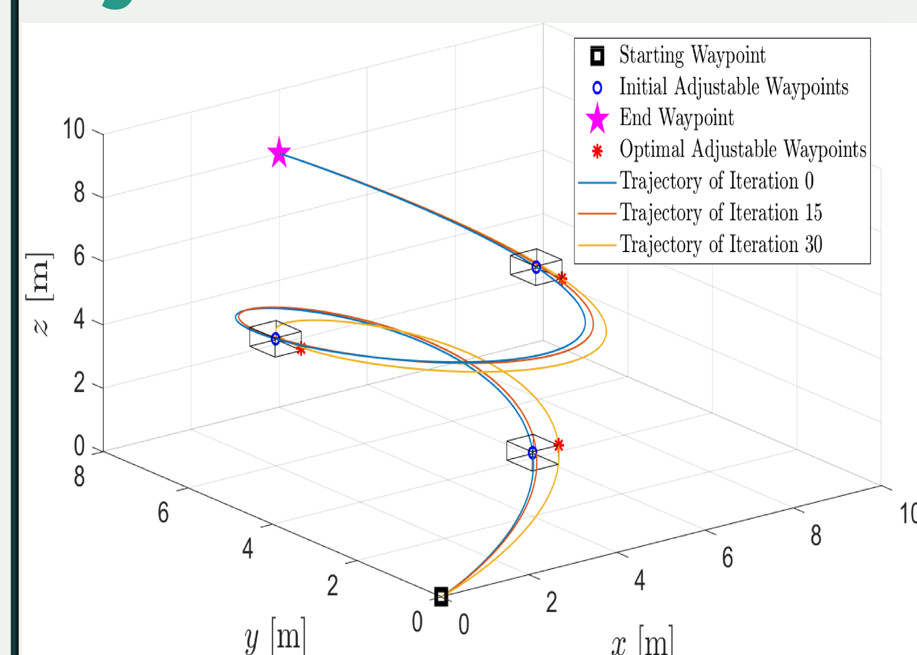


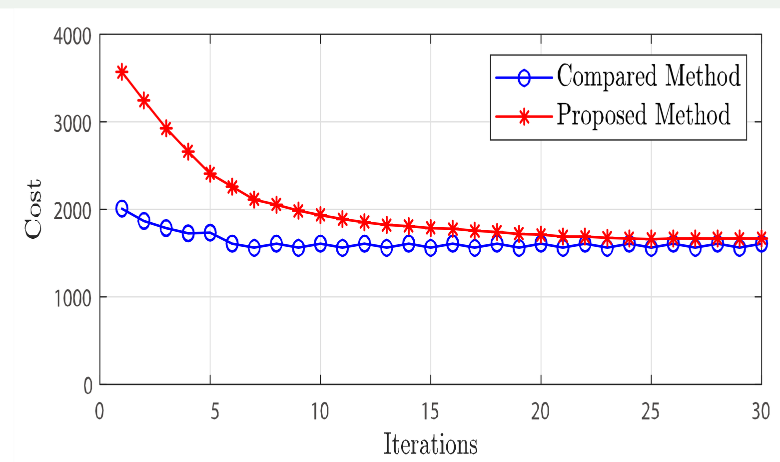
	Compared	Proposed
Optimal Cost	135.69	**132.54**
Total Computation Time [ms]	361.14	**141.31**

Case 2. Scalability Experiment with Multiple Adjustable Waypoints



Case 3. Trajectory Optimization with Dynamic Constraints





	Compared	Proposed
Optimal Cost	**1609.34**	1662.36
Total Computation Time [ms]	4342.73	**362.75**

Method

Main problem

$$\underset{\boldsymbol{\sigma},\boldsymbol{\xi},\boldsymbol{T}\in\mathcal{T}}{\text{minimize}} \quad J(\boldsymbol{\sigma},\boldsymbol{T}) = \boldsymbol{\sigma}^\top P(\boldsymbol{T})\boldsymbol{\sigma} + \boldsymbol{\sigma}^\top \boldsymbol{q}(\boldsymbol{T})$$

$$\text{subject to} \quad R\boldsymbol{\xi} \preceq \boldsymbol{s}, \quad C(\boldsymbol{T})\boldsymbol{\sigma} = \boldsymbol{\xi}, \quad A(\boldsymbol{T})\boldsymbol{\sigma} = \boldsymbol{b}.$$

Upper-level problem:
find improved temporal and spatial assignments

$$\underset{\boldsymbol{\xi}\in\mathcal{X},\boldsymbol{T}\in\mathcal{T}}{\text{minimize}} \quad J(\boldsymbol{\sigma}^*,\boldsymbol{T})$$

$$\text{subject to} \quad \boldsymbol{\sigma}^*(\boldsymbol{\xi},\boldsymbol{T}) \in \underset{\boldsymbol{\sigma}}{\text{argmin}}\{J(\boldsymbol{\sigma},\boldsymbol{T}) : \boldsymbol{\sigma} \in \mathcal{F}(\boldsymbol{\xi},\boldsymbol{T})\}.$$

Lower-level problem:
solve for the polynomial coefficients with spatial and temporal assignments obtained from the upper-level problem

$$\underset{\boldsymbol{\sigma}}{\text{minimize}} \quad J(\boldsymbol{\sigma},\boldsymbol{T})$$

$$\text{subject to} \quad C(\boldsymbol{T})\boldsymbol{\sigma} = \boldsymbol{\xi}, \quad A(\boldsymbol{T})\boldsymbol{\sigma} = \boldsymbol{b},$$

Takeaway Message

Fast UAV trajectory planning framework using reformulated bilevel optimization:

- **Simultaneously update the spatial and temporal assignment** in the upper-level problem using analytical gradients
- **Excluding the inequality constraints** in the lower-level problem to reduce the computation time.

paper: https://ieeexplore.ieee.org/document/10117594

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